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3D $\mathcal{N}=2$ massive super Yang-Mills and membranes/D2-branes in a curved background

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Abstract: We present a three dimensional novel massive $\mathcal{N}=2$ super Yang-Mills action as a low energy effective worldvolume description of the D2-branes on a pp-wave. The action contains the Myers term, mass terms for three Higgs, and terms mixing the electric fields with other two Higgs. We derive the action in three different ways, from the M-theory matrix model, from the supermembrane action, and from the Dirac-Born-Infeld action. We verify the consistent mutual agreement and comment how each approach is complementary to another. In particular, we give the eleven dimensional geometric interpretation of the vacua in the worldvolume theory as the membranes tilted to the eleventh direction with the giant gravitons around.

Keywords: D-branes, M(atrix) Theories, Penrose limit and pp-wave background, Supersymmetric Effective Theories.

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1. Introduction and summary

D-branes are a cornerstone to show that the five perturbative superstring theories in ten dimensions belong to the unique eleven dimensional theory or the M-theory [1]. Although the worldvolume action for the D-brane is generically given by the Dirac-Born-Infeld action, the precise form of its supersymmetric non-abelian generalization has not been yet known, especially in the general curved background. One can merely expect that, in the generic background, the leading term of such, if any, generalized DBI action will correspond to a certain modification of the super Yang-Mills, since in the flat background it should be the ordinary super Yang-Mills.

Recently, there have been much interests in the string/M-theory in the maximally supersymmetric ten/eleven dimensional pp-wave backgrounds. Strings on the 10D pp-wave are exactly solvable [2]–[8], and the exact form of the M-theory matrix model in the 11D pp-wave background is available now, thanks to Berenstein, Maldacena and Nastase (BMN) [2] (see also [9, 10]).

One characteristic feature of the string theory in the pp-wave background is that the string modes are all massive,

$$E_n = \sqrt{\mu^2 + \frac{n^2}{(\alpha' p_+)^2}}\,, \tag{1.1}$$

where μ is the characteristic mass parameter in the pp-wave geometry. Consequently, the worldvolume descriptions of the D-branes in the low energy limit are expected to be given by 'massive' gauge theories. It is, thus, important to understand how to realize the theory of massive vector supermultiplets while maintaining the gauge invariance [11]–[14]. The main motivation of the present paper is to construct such a massive supersymmetric gauge theory as a low energy worldvolume description of the membranes or D2-branes in the pp-wave background.

The BMN matrix model corresponds to a mass deformation of the BFSS matrix model [15]–[18], still maintaining the maximal thirty two supersymmetries. Due to the existing mass parameter, μ, the BMN matrix model presents many distinctive features, not shared by the BFSS matrix model. Among others, the supersymmetry transformations have the explicit time dependency. Accordingly the supercharges do not commute with the hamiltonian, and the corresponding supersymmetry algebra is identified as the special unitary Lie superalgebra, $\mathrm{su}(2|4;2,0)$ for $\mu > 0$ or $\mathrm{su}(2|4;2,4)$ for $\mu < 0$, of which the complexification is $A(1|3)$. Refs. [19, 20] contain the complete classification of its representations, including the quantum BPS multiplets as the 'atypical' representations. The classical counterparts of the quantum BPS states are the bosonic configurations which are the solutions of the BPS equations. In [21], all the BPS equations were obtained which correspond to the quantum BPS states preserving the various fractions of the dynamical supersymmetry, 2/16, 4/16, 8/16, 16/16. For the discussion of the perturbative aspects of the BPS states, see [22]–[24].

One characteristic feature of the generic BPS configurations is that, either they are rotating with a constant frequency, or static but curved [21] and [25]–[27]. In any case, it is an artifact of the coordinate choice that the branes, especially of the infinite size, are rotating. In fact, adopting a comoving rotating coordinate system, one can reformulate the matrix model such that the BPS configurations are static. Expanding the matrix model around the static BPS configuration leads to a non-commutative gauge theory, and taking the commutative limit one can obtain the low energy effective worldvolume action for the branes. In this way, the worldvolume action for the longitudinal five branes or the D4-branes in the pp-wave background was obtained in our previous work [11]. The resulting action is a five dimensional massive $\mathcal{N} = 1/2$ super Yang-Mills coupled to the Kähler-Chern-Simons term. In particular, the gauge fields acquire mass through the Kähler-Chern-Simons term [28].

Another interesting BPS solution found in [25], which is the main theme of the present paper, is the rotating flat membranes preserving four supersymmetries. In contrast to the longitudinal five branes or other known BPS solutions, this configuration preserves certain nontrivial four combinations of the dynamical and kinematical supersymmetries. Since the kinematical supercharges and the dynamical supercharges in the BMN matrix model have different quantum numbers for the hamiltonian, such configurations do not correspond to the energy eigenstates which have been classified in [21].

In the present paper, we study the above membrane configuration in three different ways, from the M-theory matrix model, from the supermembrane action, and from the Dirac-Born-Infeld action. We confirm the existence of the supersymmetric membrane configuration and derive its low energy effective worldvolume action in each setup. We verify the consistent mutual agreement and comment how each approach is complementary to another. In particular, after constructing the precise dual relation between the field strength and a compact scalar, we give the eleven dimensional geometric interpretation of the vacua in the worldvolume theory as a membrane tilted to the eleventh direction.

Our resulting worldvolume action is a three dimensional massive $\mathcal{N}=2$ super Yang-Mills which contains the Myers term, mass terms for three Higgs, and terms mixing the electric fields with other two Higgs. Notably the last ones make the gauge fields massive, which is quite different from the well-known mechanism through the Chern-Simons term [29]. We write the action here, as a power series of the mass parameter, μ,

$$\mathcal{S} = \frac{1}{g_{YM}^2}\int dx^3\ \mathcal{L}_0 + \mu\mathcal{L}_1 + \mu^2\mathcal{L}_2\,, \tag{1.2}$$

$$\begin{aligned}
\mathcal{L}_0 &= \mathrm{tr}_N\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\phi_a D^\mu\phi_a + \frac{1}{4}[\phi_a,\phi_b]^2 - i\frac{1}{2}\psi^\dagger\gamma^\mu D_\mu\psi - \frac{1}{2}\psi^\dagger\gamma^a[\phi_a,\psi]\Big]\,,\\
\mathcal{L}_1 &= \frac{1}{\sqrt{2}}\,\mathrm{tr}_N\Big[\frac{1}{3}(\phi_4F_{01}+\phi_3F_{02}) + \frac{1}{6}\epsilon^{pq}\phi_pD_0\phi_q - i\frac{1}{3}\epsilon^{rst}\phi_r\phi_s\phi_t + i\frac{1}{24}\psi^\dagger\Pi\psi\Big]\,,\\
\mathcal{L}_2 &= -\frac{1}{2}\left(\frac{1}{3\sqrt{2}}\right)^2\mathrm{tr}_N\left(\phi_7^2+\phi_8^2+\phi_9^2\right).
\end{aligned} \tag{1.3}$$

where $\mu,\nu=0,1,2$, $a=3,4,5,6,7,8,9$, $p=5,6$, $r=7,8,9$, $\epsilon^{56}=\epsilon^{789}=1$, and

$$\Pi = (\gamma^{14}+\gamma^{23}-\gamma^{56}+3\gamma^{789})\,. \tag{1.4}$$

The organization of the present paper and the summary of the results are as follows. In section 2, we give the basic setup for both the matrix model and the supergravity, mainly to establish the notations and the conventions. In particular, introducing the rotating coordinate system, we reformulate the BMN matrix model.

In section 3, we identify the static membrane configurations preserving four supersymmetries. In the reformulated matrix model, we explicitly construct the solution and find that the preserved supersymmetries are linear combinations of the kinematical and dynamical supersymmetries. In the supergravity setup, we perform the probe analysis and show that a membrane spanning the (x^0,x^1,x^2) directions is supersymmetric (cf. [30, 31]).

In section 4, we derive the low energy effective worldvolume actions for the M2 and D2 branes in three different ways. From the matrix model, we first get the non-commutative version of the non-abelian action, (1.2), (1.3), and then take the commutative limit. From the supermembrane action, in the low energy limit, we obtain a supersymmetric scalar action, (4.24), while from the Dirac-Born-Infeld action we acquire a bosonic massive gauge theory action, (4.30). The comparison among the results is done in the subsection 4.4. We verify the consistent mutual agreement. The latter two actions are shown to be equivalent by constructing the dual relation between the field strength and a compact scalar, (4.34).

In section 5, we identify the worldvolume supersymmetries from the matrix model and from the supergravity, respectively. In the matrix model, we first observe that in the commutative limit, transformations of some dynamical supersymmetries become singular. By imposing two constraints on the sixteen component dynamical supersymmetry parameter, the singularity is removed and only four supersymmetries survive. In the supergravity setup, the same worldvolume supersymmetries are identified as the combinations of the spacetime supersymmetry and the κ-symmetry which preserve the κ-symmetry fixing as well as the static gauge choice of the worldvolume coordinates. The subsection 5.3 presents the relevant 3D $\mathcal{N}=2$ supersymmetry algebra, (5.15), and discusses the existing three supermultiplets in the massive super Yang-Mills which are characterized by the different energy spectra. In the last subsection 5.4, we write the BPS equations of the worldvolume theory which describe the bosonic configurations preserving all the four supersymmetries. Due to the novel structure of the supersymmetry algebra, these BPS equations are not trivial. In particular, the solutions of the vacua are given by the constant fuzzy spheres formed by the last three Higgs, (ϕ^7, ϕ^8, ϕ^9) and arbitrary two vevs of (ϕ^3, ϕ^4). Utilizing the dual relation between the field strength and the compact scalar, we give the eleven dimensional geometric interpretation of the vacua. Namely they correspond to the membranes tilted to the eleventh direction with the giant gravitons around.

The appendix contains some useful formulae and explicit forms of the supercharge, R-symmetry charges and central charges in the worldvolume theory.

2. Setup

In this section, we give the basic setup for both the matrix model and the supergravity. First, we reformulate the BMN matrix model in a rotating coordinate system. In the second part, we write the basic formalism for the supermembrane action on the pp-wave.

2.1 Matrix model in a rotating coordinate system

In [25], it was shown that the BMN matrix model admits a rotating membrane preserving four supersymmetries, each of which is a linear combination of the dynamical and kinematical supersymmetries. It is an artifact of the coordinate choice that the membranes rotate with a constant frequency. The original BMN matrix model was written in a maximally symmetric coordinate system, where the pp-wave metric is of the form [32]–[34],

$$\begin{aligned} ds^2 &= -2dx^+dx^- - \Big[\left(\frac{\mu}{6}\right)^2(x_1^2+\cdots+x_6^2)+\left(\frac{\mu}{3}\right)^2(x_7^2+x_8^2+x_9^2)\Big]dx^+dx^+ + \\ &\quad + \sum_{A=1}^{9} dx^A dx^A\,, \\ F_{+789} &= \mu\,, \end{aligned} \tag{2.1}$$

with the isometry group, SO(6) $\times$ SO(3). Reformulating the matrix model in a less symmetric but 'comoving' coordinate system, one can obtain the 'static' membrane configuration. Explicitly we replace the first six coordinates, $x_1, x_2, x_3, x_4, x_5, x_6$, by the SO(2) $\times$ SO(2) $\times$ SO(2) rotating ones,

$$\begin{aligned} x_1 &\to \cos\left(\frac{\mu x^+}{6}\right) x_1 + \sin\left(\frac{\mu x^+}{6}\right) x_4\,, \qquad & x_4 &\to \cos\left(\frac{\mu x^+}{6}\right) x_4 - \sin\left(\frac{\mu x^+}{6}\right) x_1\,, \\ x_2 &\to \cos\left(\frac{\mu x^+}{6}\right) x_2 + \sin\left(\frac{\mu x^+}{6}\right) x_3\,, \qquad & x_3 &\to \cos\left(\frac{\mu x^+}{6}\right) x_3 - \sin\left(\frac{\mu x^+}{6}\right) x_2\,, \\ x_5 &\to \cos\left(\frac{\mu x^+}{6}\right) x_5 - \sin\left(\frac{\mu x^+}{6}\right) x_6\,, \qquad & x_6 &\to \cos\left(\frac{\mu x^+}{6}\right) x_6 + \sin\left(\frac{\mu x^+}{6}\right) x_5\,, \end{aligned} \tag{2.2}$$

so that the metric of the eleven dimensional pp-wave background, (2.1), is, in the new coordinate system, of the form

$$\begin{aligned} ds^2 = &-2dx^+dx^- - \frac{\mu}{3}(x_1dx_4 - x_4dx_1 + x_2dx_3 - x_3dx_2 - x_5dx_6 + x_6dx_5)dx^+ - \\ &- \left(\frac{\mu}{3}\right)^2 (x_7^2 + x_8^2 + x_9^2)dx^+dx^+ + \sum_{A=1}^{9} dx^A dx^A\,. \end{aligned} \tag{2.3}$$

The rotation of the $(5,6)$ plane is not necessary to obtain the static configuration. However, it makes the supercharges commute with the hamiltonian in the worldvolume theory, as is the case for the worldvolume theory of the longitudinal five branes [11].

The corresponding M-theory matrix model on this background is obtained from the original BMN matrix model by incorporating the above time dependent rotations. With $t \equiv x^+$, the transformations of the bosons are essentially the same as above,

$$X_1 \to \cos\left(\frac{\mu t}{6}\right) X_1 + \sin\left(\frac{\mu t}{6}\right) X_4\,, \qquad \text{etc.} \tag{2.4}$$

while those of the fermions read, from the standard Lorentz transformation rule,

$$\psi \;\to\; e^{\mu/12(\gamma^{14}+\gamma^{23}-\gamma^{56})t}\psi\,. \tag{2.5}$$

The modified, but nevertheless equivalent, M-theory matrix model on the fully supersymmetric pp-wave background spells[1] with a mass parameter, μ,

$$\mathcal{S} = \frac{l_p^6}{R^3}\int dt\; \mathcal{L}_0 + \mu\mathcal{L}_1 + \mu^2\mathcal{L}_2\,, \tag{2.6}$$

$$\mathcal{L}_0 = \mathrm{Tr}\left(\frac{1}{2}D_tX^AD_tX_A + \frac{1}{4}[X^A,X^B]^2 + i\frac{1}{2}\psi^\dagger D_t\psi - \frac{1}{2}\psi^\dagger\gamma^A[X_A,\psi]\right)\,, \tag{2.7}$$

$$\mathcal{L}_1 = \mathrm{Tr}\left[-\frac{1}{6}J^{ab}X_aD_tX_b - i\frac{1}{3}\epsilon^{rst}X_rX_sX_t + i\frac{1}{24}\psi^\dagger(\gamma^{14}+\gamma^{23}-\gamma^{56}+3\gamma^{789})\psi\right]\,, \tag{2.8}$$

$$\mathcal{L}_2 = -\frac{1}{2}\left(\frac{1}{3}\right)^2 \mathrm{Tr}\big(X_7^2 + X_8^2 + X_9^2\big)\,, \tag{2.9}$$

[1] For the derivation of the original BMN matrix model either from the supergraviton action or from the Polyakov type supermembrane action, we refer [2] and [22] respectively.

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where $a, b = 1, 2, 3, 4, 5, 6, \;\; r, s, t = 7, 8, 9, \;\; A, B = 1, 2, \ldots, 9$ and J^{ab} is a skew-symmetric 6×6 constant two form of which the non-vanishing components are $J^{14} = J^{23} = J^{65} = 1$ only, up to the anti-symmetric property. In the present paper, we adopt generic euclidean nine dimensional gamma matrices, $\gamma^A = (\gamma^A)^\dagger$, $\gamma^{12\cdots 9} = 1$. Namely we do not adopt the usual real and symmetric Majorana representation. Accordingly there exits a nontrivial 16×16 charge conjugation matrix, C,

$$(\gamma^A)^T = (\gamma^A)^* = C^{-1}\gamma^A C\,, \qquad C = C^T = (C^\dagger)^{-1}\,. \tag{2.10}$$

The spinors, ψ, satisfy the Majorana condition leaving eight independent complex components,

$$\psi = C\psi^*\,. \tag{2.11}$$

The covariant derivatives are in our convention, $D_t\mathcal{O} = \frac{d}{dt}\mathcal{O} - i[A_0, \mathcal{O}]$ so that X and A_0 are of the mass dimension one, while ψ has the mass dimension $3/2$. Compared to the original BMN matrix model, the quadratic mass terms for the first six bosonic coordinates are absent.

The dynamical or linearly realized supersymmetry transformations are

$$\delta A_0 = i\psi^\dagger \mathcal{E}(t)\,, \qquad \delta X^A = i\psi^\dagger\gamma^A\mathcal{E}(t)\,, \tag{2.12}$$

$$\begin{aligned}\delta\psi = \Big[&D_t X^A\gamma_A - i\frac{1}{2}[X^A, X^B]\gamma_{AB} - \frac{\mu}{3}(X^7\gamma_7 + X^8\gamma_8 + X^9\gamma_9)\gamma^{789} + \\ &+ \frac{\mu}{6}(X^1\gamma_1 + X^4\gamma_4)(\gamma^{789} - \gamma^{14}) + \frac{\mu}{6}(X^2\gamma_2 + X^3\gamma_3)(\gamma^{789} - \gamma^{23}) + \\ &+ \frac{\mu}{6}(X^5\gamma_5 + X^6\gamma_6)(\gamma^{789} + \gamma^{56})\Big]\mathcal{E}(t)\,,\end{aligned} \tag{2.13}$$

where

$$\mathcal{E}(t) = e^{\mu/12(-\gamma^{14} - \gamma^{23} + \gamma^{56} + \gamma^{789})t}\mathcal{E}\,, \qquad \mathcal{E} = C\mathcal{E}^*\,, \tag{2.14}$$

and $\mathcal{E}$ is an arbitrary sixteen component constant spinor.

In addition, there is the kinematical or the non-linearly realized supersymmetry,

$$\delta A_0 = \delta X^A = 0\,, \qquad \delta\psi = e^{-\mu/12(\gamma^{14} + \gamma^{23} - \gamma^{56} + 3\gamma^{789})t}\mathcal{E}'\,, \qquad \mathcal{E}' = C\mathcal{E}'^*\,. \tag{2.15}$$

2.2 Supermembrane action on the pp-wave

Here we briefly review the formalism of the supermembrane action given in [35]. Mostly following the conventions therein, except $\epsilon^{012} = 1$, we denote the curved space indices by $\tilde{M} = (M, \alpha)$, and the tangent space indices by $\tilde{A} = (\hat{R}, \hat{a})$, while $\mu, \nu = 0, 1, 2$ are the worldvolume indices of the supermembrane.

Using the superspace embedding coordinates,[2] $Z^{\tilde{M}}(\xi) = (x^M(\xi), \theta^\alpha(\xi))$, the supermembrane action is given by

$$S_{M2} = -T_{M2}\int d\xi^3\sqrt{-h(Z(\xi))} + T_{M2}\int B\,. \tag{2.16}$$

[2] θ is a 32-component spinor satisfying the Majorana condition, (A.4).

Here h is the determinant of the induced worldvolume metric,

$$h = \det h_{\mu\nu}\,, \qquad h_{\mu\nu} = \Pi_\mu{}^{\hat R}\Pi_\nu{}^{\hat S}\eta_{\hat R\hat S}\,, \tag{2.17}$$

written in terms of the pull-back, $\Pi_\mu{}^{\tilde A}$, of the supervielbein, $E^{\tilde A}_{\tilde M}$,

$$\Pi_\mu{}^{\tilde A} = \partial_\mu x^M E^{\tilde A}_M + \partial_\mu\theta^\alpha E^{\tilde A}_\alpha\,. \tag{2.18}$$

The three-form superfield, B, gives the Wess-Zumino term,

$$T_{M2}\int B = T_{M2}\int d\xi^3\frac{\epsilon^{\mu\nu\rho}}{6\sqrt{-h}}\Pi_\mu{}^{\tilde A}\Pi_\nu{}^{\tilde B}\Pi_\rho{}^{\tilde C}B_{\tilde C\tilde B\tilde A}\,, \tag{2.19}$$

and can be expanded in terms of θ and $\bar\theta = i\theta^\dagger\Gamma^0$ [36],

$$B = -\frac{1}{6}e^{\hat R}\wedge e^{\hat S}\wedge e^{\hat U}C_{\hat R\hat S\hat U} + \frac{1}{2}e^{\hat R}\wedge e^{\hat S}\wedge\bar\theta\Gamma_{\hat R\hat S}D\theta + \mathcal{O}(\theta^4)\,, \tag{2.20}$$

where

$$D\theta = d\theta - \frac{1}{4}\omega^{\hat R\hat S}\Gamma_{\hat R\hat S}\theta + e^{\hat R}T^{NPQR}_{\hat R}F_{NPQR}\theta\,, \qquad T^{NPQR}_M = \frac{1}{288}\left(\Gamma_M^{\ NPQR} - 8\delta_M^{[N}\Gamma^{PQR]}\right). \tag{2.21}$$

The κ-symmetry of the supermembrane action is given by

$$\delta_\kappa Z^{\tilde M}E^{\hat R}_{\tilde M} = 0\,, \qquad \delta_\kappa Z^{\tilde M}E^{\hat a}_{\tilde M} = (1+\Gamma)^{\hat a}{}_{\hat b}\kappa^{\hat b}\,, \tag{2.22}$$

where $\kappa(\xi)$ is an arbitrary local fermionic parameter and Γ is the projection matrix,

$$\Gamma = \frac{1}{6}\frac{\epsilon^{\mu\nu\rho}}{\sqrt{-h}}\Pi_\mu{}^{\hat R}\Pi_\nu{}^{\hat S}\Pi_\rho{}^{\hat U}\Gamma_{\hat R\hat S\hat U}\,, \qquad \epsilon^{012} = 1\,, \tag{2.23}$$

satisfying

$$\mathrm{tr}\,\Gamma = 0\,, \qquad \Gamma^2 = 1\,. \tag{2.24}$$

The component form of the supermembrane action in the general background is known only up to θ^2 order [35]. However, the explicit forms in the maximally supersymmetric $AdS_4\times S^7$ and $AdS_7\times S^4$ have been determined to all orders, using the coset method [36]. The corresponding supervielbein for these spaces is

$$E \,=\, D\theta + \sum_{n=1}^{16}\frac{1}{(2n+1)!}\mathcal{M}^{2n}D\theta\,, \tag{2.25}$$

$$E^{\hat R} \,=\, e^{\hat R} + \bar\theta\Gamma^{\hat R}D\theta + 2\sum_{n=1}^{15}\frac{1}{(2n+2)!}\bar\theta\Gamma^{\hat R}\mathcal{M}^{2n}D\theta\,, \tag{2.26}$$

where

$$\begin{aligned}(\mathcal{M}^2)^{\hat a}{}_{\hat b} \,=&\, 2(T^{NPQR}_M\theta)^{\hat a}F_{NPQR}(\bar\theta\Gamma^M)_{\hat b} - \\ &- \frac{1}{288}(\Gamma_{MN}\theta)^{\hat a}\left[\bar\theta(\Gamma^{MNPQRS}F_{PQRS} + 24F^{MNPQ}\Gamma_{PQ})\right]_{\hat b}\,.\end{aligned} \tag{2.27}$$

Since the maximally supersymmetric pp-wave can be obtained by taking a Penrose limit of the maximally supersymmetric $AdS \times S$ spaces, the result above is still valid for the pp-wave geometry [22].

Rotating x^1, x^2, x^3, x^4 coordinates as in (2.2) and transforming the x^- coordinate as[3] $x^- \to x^- - (\mu/6)(x^1x^4 + x^2x^3)$, we rewrite the pp-wave geometry (2.1), with $x^{\pm} = \frac{1}{\sqrt{2}}(t \pm y)$,

$$\begin{aligned} ds_{11}^2 &= -(1+H/2)\left[dt + \frac{H/2}{1+H/2}dy\right]^2 + \frac{1}{1+H/2}dy^2 + \left[dx^1 + \frac{\mu}{3\sqrt{2}}x^4(dt+dy)\right]^2 + \\ &\quad + \left[dx^2 + \frac{\mu}{3\sqrt{2}}x^3(dt+dy)\right]^2 + \sum_{l=3}^{9} dx^l dx^l \,, \\ F_{t789} &= F_{y789} = \frac{\mu}{\sqrt{2}} \,, \end{aligned} \tag{2.28}$$

where

$$H = \left(\frac{\mu}{6}\right)^2 (x_5^2 + x_6^2) + \left(\frac{\mu}{3}\right)^2 (x_3^2 + x_4^2 + x_7^2 + x_8^2 + x_9^2) \,. \tag{2.29}$$

Appendix A.1 contains the explicit forms of the bosonic vielbein and spin connections as well as our choice of the 11D gamma matrix representation which utilizes the 9D gamma matrix used in the M-theory matrix model setup.

3. BPS membranes preserving four supersymmetries

In this section, we discuss the existence of the BPS membrane configurations which preserve four supersymmetries, in each framework. First, in the matrix model setup, we obtain the static BPS membrane solution, and show that only four supersymmetries are unbroken. They are given by the linear combination of the dynamical and kinematical supersymmetries. Then, in the supergravity setup, we perform the relevant probe analysis to identify the corresponding membrane configuration and the four supersymmetries.

3.1 Matrix model analysis

We consider the following static flat membrane configurations,

$$X_1 = i\hat{\partial}_1 \,, \qquad X_2 = i\hat{\partial}_2 \,, \qquad A_0 = X_A = 0 \,, A = 3, 4, \ldots, 9 \,. \tag{3.1}$$

Here the operators, $\hat{\partial}_i$'s are related to the coordinates of a non-commutative plane,

$$x^1 = i\theta\hat{\partial}_2 \,, \qquad x^2 = -i\theta\hat{\partial}_1 \,, \tag{3.2}$$

such that

$$[X_1, X_2] = i\frac{1}{\theta} \,, \qquad [x^1, x^2] = i\theta \,, \qquad [\hat{\partial}_i, x^j] = \delta_i{}^j \,. \tag{3.3}$$

This relation gives a set of harmonic oscillators, and the most general irreducible representation is specified by the superselection rule on the number of the ground states which we

[3] Note that the shift of x^- coordinate would result in adding a total derivative term in the M-theory matrix model.

denote by N. Thus, the Hilbert space, $\mathcal{H}$, on which the infinite matrices act decomposes as a direct product of a harmonic oscillator Hilbert space, $H_{h.o.}$ and an N dimensional vector space, V_N,

$$\mathcal{H} = H_{h.o.} \otimes V_N \,. \tag{3.4}$$

Explicitly, using the bra and ket notations, one can regroup the states in the Hilbert space [37],

$$|n, s\rangle \,, \qquad n = 0, 1, \ldots, \infty \,, \qquad s = 1, 2, \ldots, N \,, \tag{3.5}$$

so that the creation and annihilation operators are respectively,

$$\sum_{n,s} \sqrt{n+1} |n+1, s\rangle\langle n, s| \,, \qquad \sum_{n,s} \sqrt{n+1} |n, s\rangle\langle n+1, s| \,. \tag{3.6}$$

Of course, this represents N parallel membranes which, we show, preserve four supersymmetries.

To see that the configuration preserves four supersymmetries, we pay attention to the supersymmetry transformation of the fermions which, in the present case, reduces to

$$\delta\psi = \frac{1}{\theta}\gamma^{12} e^{\mu/12(-\gamma^{14}-\gamma^{23}+\gamma^{56}+\gamma^{789})t}\mathcal{E} + e^{-\mu/12(\gamma^{14}+\gamma^{23}-\gamma^{56}+3\gamma^{789})t}\mathcal{E}' \,, \tag{3.7}$$

where the first and second parts are dynamical and kinematical supersymmetry transformations respectively. Requiring it to vanish, we obtain

$$\mathcal{E}' = -\frac{1}{\theta}\gamma_{12} e^{\mu/6(-\gamma^{14}-\gamma^{23}+2\gamma^{789})t}\mathcal{E} \,. \tag{3.8}$$

Since the left hand side is time independent, the Killing spinor, $\mathcal{E}$, must satisfy

$$(\gamma^{14} + \gamma^{23} - 2\gamma^{789})\mathcal{E} = 0 \,. \tag{3.9}$$

By multiplying $\gamma^{1234}, \gamma^{1456}, \gamma^{2356}$ to the left and using $\gamma^{12\cdots 9} = 1$, one can show that the constraint is actually equivalent to

$$\gamma^{14}\mathcal{E} = \gamma^{23}\mathcal{E} = \gamma^{56}\mathcal{E} = \gamma^{789}\mathcal{E} \,. \tag{3.10}$$

In a more concise form, this is again equivalent to

$$\Omega\mathcal{E} = \mathcal{E} \,, \qquad \Omega = \frac{1}{4}\left(1 - \gamma^{1234} - \gamma^{1456} - \gamma^{2356}\right) . \tag{3.11}$$

Ω is a projection matrix for the Killing spinors [21, 38] satisfying,

$$\Omega^\dagger = \Omega \,, \qquad C\Omega^* C^{-1} = \Omega \,, \qquad \Omega^2 = \Omega \,, \qquad \mathrm{tr}\,\Omega = 4 \,, \tag{3.12}$$

and, to agree with (3.10),

$$\gamma^{14}\Omega = \gamma^{23}\Omega = \gamma^{56}\Omega = \gamma^{789}\Omega \,. \tag{3.13}$$

Thus, the configuration preserves four supersymmetries, each of which is a linear combination of the dynamical and kinematical supersymmetries given by

$$\mathcal{E}' = -\frac{1}{\theta}\gamma_{12}\mathcal{E} \,. \tag{3.14}$$

In the ordinary BFSS matrix model or the $\mu = 0$ flat background case, the same membrane configuration, (3.1) and (3.3), preserves sixteen supersymmetries out of thirty two. They are given by the linear combinations of the kinematical and dynamical supersymmetries, (3.14), without any constraint on $\mathcal{E}$.

Finally, from (2.13), it is worth to note that the membrane configuration can be shifted to the third and fourth directions still preserving the four supersymmetries,

$$X^3 = c^3\, 1\,, \qquad X^4 = c^4\, 1\,. \tag{3.15}$$

In the subsection 5.4, we will see that this configuration, in fact, corresponds to membranes tilted to the 11th direction.

3.2 Probe analysis

Here we count the number of supersymmetries a M2-brane probe preserves. In the probe analysis, we only consider the rigid flat M2-brane. We will take into account its fluctuations when we consider the worldvolume supersymmetry in the subsection 5.2.

The supersymmetry variation of the gravitino, ψ_M, is

$$\delta_\eta \psi_M = \left(\partial_M - \frac{1}{4}\omega_M^{\hat{R}\hat{S}}\Gamma_{\hat{R}\hat{S}} + T_M^{NPQR} F_{NPQR} \right) \eta\,. \tag{3.16}$$

The Killing spinor, η, satisfying $\delta_\eta \psi = 0$, for the given pp-wave geometry, (2.28), is of the form,

$$\eta = \left(\cosh \ln \left(1 + \frac{H}{2}\right)^{1/4} + \Gamma^{\hat{+}\hat{-}} \sinh \ln \left(1 + \frac{H}{2}\right)^{1/4} \right) \left(1 - \sum_{n=1}^{9} x^n \Omega_n \right) e^{-x^+ \Omega_+} \eta_0\,, \tag{3.17}$$

where $\Gamma^{\hat{\pm}} = \frac{1}{\sqrt{2}}(\Gamma^{\hat{t}} \pm \Gamma^{\hat{y}})$, η_0 is an arbitrary 32-component constant spinor, and

$$\Omega_+ = \frac{\mu}{12}(\Gamma^{\hat{+}\hat{-}\hat{7}\hat{8}\hat{9}} + \Gamma^{\hat{1}\hat{4}} + \Gamma^{\hat{2}\hat{3}} - 2\Gamma^{\hat{7}\hat{8}\hat{9}})\,, \tag{3.18}$$

$$\Omega_n = \begin{cases} \frac{\mu}{12}\Gamma^{\hat{+}\hat{4}}(1 - \Gamma^{\hat{7}\hat{8}\hat{9}\hat{4}\hat{1}})\,, & n = 1 \\ \frac{\mu}{12}\Gamma^{\hat{+}\hat{3}}(1 - \Gamma^{\hat{7}\hat{8}\hat{9}\hat{3}\hat{2}})\,, & n = 2 \\ \frac{\mu}{12}\Gamma^{\hat{+}\hat{2}}(1 + \Gamma^{\hat{7}\hat{8}\hat{9}\hat{3}\hat{2}})\,, & n = 3 \\ \frac{\mu}{12}\Gamma^{\hat{+}\hat{1}}(1 + \Gamma^{\hat{7}\hat{8}\hat{9}\hat{4}\hat{1}})\,, & n = 4 \\ \frac{\mu}{24}\Gamma^{\hat{+}}(\Gamma^{\hat{n}}\Gamma^{\hat{7}\hat{8}\hat{9}} + 3\Gamma^{\hat{7}\hat{8}\hat{9}}\Gamma^{\hat{n}})\,, & n = 5,6,7,8,9. \end{cases} \tag{3.19}$$

The unbroken supersymmetries of the M2-brane probe are given by the Killing spinors satisfying [39]

$$\Gamma \eta = \eta\,. \tag{3.20}$$

In particular, in our supergravity setup, we consider a single M2-brane which spans the (t, x^1, x^2) directions while being located at the origin of the transverse coordinates, $x^L = 0$, $L = 3, \ldots, 10$. Taking the static gauge,

$$t = \xi^0\,, \qquad x^1 = \xi^1\,, \qquad x^2 = \xi^2\,, \tag{3.21}$$

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the projection matrix, Γ, becomes

$$\Gamma = \Gamma_{\hat{t}\hat{1}\hat{2}}\,, \tag{3.22}$$

so that the unbroken supersymmetry condition, $\Gamma\eta = \eta$, reduces to

$$\Gamma_{\hat{t}\hat{1}\hat{2}}\eta_0 = \eta_0\,, \qquad [\Gamma_{\hat{t}\hat{1}\hat{2}},\,\Omega_+]\eta_0 = 0\,, \qquad [\Gamma_{\hat{t}\hat{1}\hat{2}},\,\Omega_1]\eta_0 = 0\,, \qquad [\Gamma_{\hat{t}\hat{1}\hat{2}},\,\Omega_2]\eta_0 = 0\,. \tag{3.23}$$

From the explicit form of $\Omega_+, \Omega_{1,2}$, these conditions are, at last, equivalent to

$$\Gamma_{\hat{t}\hat{1}\hat{2}}\eta_0 = \Gamma^{\hat{7}\hat{8}\hat{9}\hat{4}\hat{1}}\eta_0 = \Gamma^{\hat{7}\hat{8}\hat{9}\hat{3}\hat{2}}\eta_0 = \eta_0\,, \tag{3.24}$$

which shows that the probe configuration preserves four supersymmetries.

4. Derivation of the 3D massive super Yang-Mills action

In this section, we derive explicitly the novel three dimensional massive $\mathcal{N} = 2$ super Yang-Mills action, (1.2), in three different ways, one from the matrix model and the other two from the supermembrane action and the D2-brane Dirac-Born-Infeld action. In the matrix model setup, we derive the full non-abelian action, while in the M2 and D2 setups, we identify the abelian part. All the results we obtain here are consistent.

4.1 Matrix model derivation

By expanding the matrix model around the above supersymmetric coincident N membranes, we derive the massive super Yang-Mills action. To do so, we introduce the gauge fields as the longitudinal fluctuations around the membranes, and write

$$\begin{aligned} [b]X_i &= i\hat{\partial}_i + A_i\,, \qquad & i &= 1,2\,, \\ X_a &= \phi_a\,, \qquad & a &= 3,4,5,6,7,8,9\,. \end{aligned} \tag{4.1}$$

Consequently

$$\begin{aligned} D_t X_i &= F_{0i}\,, \qquad & [X_1, X_2] &= i(F_{12} + \theta^{-1})\,, \\ [X_i, \phi] &= iD_i\phi\,, \qquad & [X_i, \psi] &= iD_i\psi\,, \end{aligned} \tag{4.2}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, $\mu,\nu = 0,1,2$ and the derivative of a function along the non-commutative coordinate is, from (3.3), $\partial_i\phi = [\hat{\partial}_i, \phi]$. The fields have the standard gauge transformation properties,

$$A_\mu \;\rightarrow\; UA_\mu U^\dagger + iU\partial_\mu U^\dagger\,, \qquad \phi \;\rightarrow\; U\phi U^\dagger\,. \tag{4.3}$$

To write the matrix model, (2.6), in terms of the gauge fields, we first note

$$J^{lm}\,\mathrm{Tr}(X_l D_t X_m) = -2\,\mathrm{Tr}(\phi_4 F_{01} + \phi_3 F_{02}) + \frac{d}{dt}\,\mathrm{Tr}(X_1X_4 + X_2X_3)\,, \qquad l,m = 1,2,3,4\,. \tag{4.4}$$

Utilizing the fact that the trace over the Hilbert space, $\mathcal{H}$, can decompose into the integration over the non-commutative plane and the trace over the "U(N)" indices,

$$\mathrm{Tr}\,\mathcal{O}(x) = \frac{1}{2\pi\theta}\int dx^2\;\mathrm{tr}_N\,\mathcal{O}(x)\,, \tag{4.5}$$

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one can rewrite the matrix model as a non-commutative action. After discarding the total derivative terms and the mass of the membranes, our matrix model, (2.6), in the membrane background leads to a non-commutative massive super Yang-Mills,

$$\mathcal{S} = \frac{l_p^6}{2\pi\theta R^3}\int dx^3\ \mathcal{L}_0 + \mu\mathcal{L}_1 + \mu^2\mathcal{L}_2\,, \tag{4.6}$$

$$\mathcal{L}_0 = \mathrm{tr}_N\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D_\mu\phi_a D^\mu\phi_a + \frac{1}{4}[\phi_a,\phi_b]^2 - i\frac{1}{2}\psi^\dagger\gamma^\mu D_\mu\psi - \frac{1}{2}\psi^\dagger\gamma^a[\phi_a,\psi]\right], \tag{4.7}$$

$$\mathcal{L}_1 = \mathrm{tr}_N\left[\frac{1}{3}(\phi_4F_{01}+\phi_3F_{02}) + \frac{1}{6}\epsilon^{pq}\phi_p D_0\phi_q - i\frac{1}{3}\epsilon^{rst}\phi_r\phi_s\phi_t + i\tfrac{1}{24}\psi^\dagger\Pi\psi\right], \tag{4.8}$$

$$\mathcal{L}_2 = -\frac{1}{2}\left(\frac{1}{3}\right)^2 \mathrm{tr}_N\left(\phi_7^2+\phi_8^2+\phi_9^2\right), \tag{4.9}$$

where $\mu = 0,1,2,\ a = 3,4,5,6,7,8,9,\ p = 5,6,\ r = 7,8,9,\ \epsilon^{56} = \epsilon^{789} = 1$, and

$$\Pi = (\gamma^{14} + \gamma^{23} - \gamma^{56} + 3\gamma^{789})\,. \tag{4.10}$$

Any product is to be understood as the non-commutative star product. The dynamical supersymmetry transformations are, from (2.13),

$$\delta A_\mu = i\psi^\dagger\gamma_\mu\mathcal{E}(t)\,, \qquad \delta\phi_a = i\psi^\dagger\gamma_a\mathcal{E}(t)\,, \tag{4.11}$$

$$\delta\psi = \begin{bmatrix} \frac{1}{2}F_{\mu\nu}\tilde{\gamma}^\mu\gamma^\nu + D_\mu\phi_a\tilde{\gamma}^\mu\gamma^a - i\frac{1}{2}[\phi_a,\phi_b]\gamma^{ab} - \frac{\mu}{3}(\phi_7\gamma^7+\phi_8\gamma^8+\phi_9\gamma^9)\gamma^{789}+ \\ +\frac{1}{\theta}\gamma^{12} + \frac{\mu}{6}\left((-\frac{1}{\theta}x^2+A_1)\gamma^1+\phi_4\gamma^4\right)(\gamma^{789}-\gamma^{14})+ \\ +\frac{\mu}{6}\left((\frac{1}{\theta}x^1+A_2)\gamma^2+\phi_3\gamma^3\right)(\gamma^{789}-\gamma^{23}) + \frac{\mu}{6}(\phi_5\gamma^5+\phi_6\gamma^6)(\gamma^{789}+\gamma^{56}) \end{bmatrix}\mathcal{E}(t)\,,$$

where $a = 3,4,5,6,7,8,9$ and

$$\mathcal{E}(t) = e^{\mu/12(-\gamma^{14}-\gamma^{23}+\gamma^{56}+\gamma^{789})t}\mathcal{E}\,, \qquad \mathcal{E} = C\mathcal{E}^*\,. \tag{4.12}$$

Note that the full supersymmetry remains unbroken for this reformulation, which is no surprise as the non-commutative three dimensional action (4.6) is merely a particular manifestation of the background independent M-theory matrix model [40, 37].

Despite the similarity between the terms mixing the field strength with the Higgs and the Chern-Simons term, there is no quantization rule for the coefficient, contrary to the Chern-Simons theory on a non-commutative plane [41], since the terms here are manifestly gauge invariant.

By taking the commutative limit, $\theta \to 0$ while keeping the coupling constant, $2\pi\theta R^3/l_p^6$, fixed, one can obtain a commutative action, which is exactly of the same form as (1.2), but μ therein is replaced by $\sqrt{2}\mu$. This $\sqrt{2}$ factor can be absorbed by scaling the worldvolume coordinates and redefining the field variables as

$$(x^\mu,\ A_\mu,\ \phi_a,\ \psi) \ \longrightarrow\ (2^{-1/2}x^\mu,\ 2^{1/2}A_\mu,\ 2^{1/2}\phi_a,\ 2^{3/4}\psi)\,. \tag{4.13}$$

This scaling will be justified in the subsection 4.4.

In the commutative limit, some supersymmetries become singular and broken. In the subsection 5.1, we will show that only four supersymmetries survive.

4.2 Derivation from the supermembrane action

In this subsection, we obtain the low energy effective worldvolume action for the membrane spanning the (x^0, x^1, x^2) directions from the supermembrane action. What we mean by "low energy" is the following limits. We scale the M2-brane tension as $T_{M2} = 1/(4\pi^2 l_p^3) \sim \epsilon^{-2} \to \infty$, and let the transverse coordinates $x^l, y = x^{10}$, $l = 3, \ldots, 9$, the fermionic superpartner, θ, scale like $(x^l, y, \theta) \sim \epsilon$. This, after the compactification along the y direction, corresponds to the scaling of the string length and the string coupling as $l_s \sim g_s \sim \epsilon^{1/2} \to 0$, keeping $g_{YM}^2 = g_s/l_s$ finite.

In the above scaling limits, $\mathcal{M}^2 = \mathcal{O}(\theta^2) \sim \epsilon^2$ and, from (2.25) and (2.26), [35, 36]

$$
\begin{aligned}
E_\alpha^{\hat{R}} &= -(\bar{\theta}\Gamma^{\hat{R}})_\alpha + \mathcal{O}(\epsilon^3)\,,\\
E_M^{\hat{R}} &= e_M^{\hat{R}} + \bar{\theta}\Gamma^{\hat{R}}\Big(-\frac{1}{4}\omega_M^{\hat{R}\hat{S}}\Gamma_{\hat{R}\hat{S}} + T_M^{NPQR}F_{NPQR}\Big)\theta + \mathcal{O}(\epsilon^3)\,,\\
E_\alpha^{\hat{a}} &= \delta_\alpha^{\hat{a}} + \mathcal{O}(\epsilon^2)\,,\\
E_M^{\hat{a}} &= \Big[\Big(-\frac{1}{4}\omega_M^{\hat{R}\hat{S}}\Gamma_{\hat{R}\hat{S}} + T_M^{NPQR}F_{NPQR}\Big)\theta\Big]^{\hat{a}} + \mathcal{O}(\epsilon^2)\,.
\end{aligned}
\tag{4.14}
$$

Thus,

$$
\begin{aligned}
\Pi_\mu{}^{\hat{R}} &= \partial_\mu x^M e_M^{\hat{R}} + \partial_\mu x^M \bar{\theta}\Gamma^{\hat{R}}\Big(-\frac{1}{4}\omega_M^{\hat{S}\hat{U}}\Gamma_{\hat{S}\hat{U}} + T_M^{NPQR}F_{NPQR}\Big)\theta + \bar{\theta}\Gamma^{\hat{R}}\partial_\mu\theta + \mathcal{O}(\epsilon^3)\,,\\
\Pi_\mu{}^{\hat{a}} &= \partial_\mu\theta^{\hat{a}} + \partial_\mu x^M\Big[\Big(-\frac{1}{4}\omega_M^{\hat{R}\hat{S}}\Gamma_{\hat{R}\hat{S}} + T_M^{NPQR}F_{NPQR}\Big)\theta\Big]^{\hat{a}} + \mathcal{O}(\epsilon^2)\,.
\end{aligned}
$$

Adopting the static gauge (3.21) and letting

$$
\Omega_t = \Omega_0 \equiv \frac{1}{\sqrt{2}}\Omega_+ = \frac{\mu}{12\sqrt{2}}(\Gamma^{\hat{+}\hat{-}\hat{7}\hat{8}\hat{9}} + \Gamma^{\hat{1}\hat{4}} + \Gamma^{\hat{2}\hat{3}} - 2\Gamma^{\hat{7}\hat{8}\hat{9}})\,,
\tag{4.15}
$$

the three form superfield becomes[4]

$$
B = d\xi^\mu \wedge d\xi^\nu \wedge d\xi^\rho\Big(\frac{1}{2}\bar{\theta}\Gamma_{\hat{\mu}\hat{\nu}}D_\rho\theta\Big) + \mathcal{O}(\epsilon^3)\,,
\tag{4.16}
$$

while the worldvolume induced metric is explicitly, with $l = 3, \ldots, 9$,

$$
\begin{aligned}
h_{00} &= -\left(1 + \frac{\tilde{H}}{2}\right) + (\partial_0 y)^2 + (\partial_0 x^l)^2 - 2\bar{\theta}\Gamma^{\hat{t}}(\partial_0\theta + \Omega_t\theta) + \mathcal{O}(\epsilon^3)\,,\\
h_{11} &= 1 + (\partial_1 y)^2 + (\partial_1 x^l)^2 + \frac{2\mu}{3\sqrt{2}}x^4\partial_1 y + 2\bar{\theta}\Gamma^{\hat{1}}(\partial_1\theta + \Omega_1\theta) + \mathcal{O}(\epsilon^3)\,,\\
h_{22} &= 1 + (\partial_2 y)^2 + (\partial_2 x^l)^2 + \frac{2\mu}{3\sqrt{2}}x^3\partial_2 y + 2\bar{\theta}\Gamma^{\hat{2}}(\partial_2\theta + \Omega_2\theta) + \mathcal{O}(\epsilon^3)\,,\\
h_{01} &= \frac{\mu}{3\sqrt{2}}x^4 + \mathcal{O}(\epsilon^2)\,, \qquad h_{02} = \frac{\mu}{3\sqrt{2}}x^3 + \mathcal{O}(\epsilon^2)\,, \qquad h_{12} = \mathcal{O}(\epsilon^2)\,.
\end{aligned}
\tag{4.17}
$$

Hence,

$$
-\det h_{\mu\nu} = 1 + \frac{H}{2} + \partial_\mu y\partial^\mu y + \partial_\mu x^l\partial^\mu x^l + \frac{2\mu}{3\sqrt{2}}(x^4\partial_1 y + x^3\partial_2 y) + 2\bar{\theta}\Gamma^{\hat{\mu}}(\partial_\mu + \Omega_\mu)\theta + \mathcal{O}(\epsilon^3)\,.
\tag{4.18}
$$

[4]Note that the first term in (2.20) scales as ϵ^3 in our setup.

In the above, we have introduced

$$\tilde{H}=H-\left(\frac{\mu}{3}\right)^2(x_3^2+x_4^2)=\left(\frac{\mu}{6}\right)^2(x_5^2+x_6^2)+\left(\frac{\mu}{3}\right)^2(x_7^2+x_8^2+x_9^2)\,. \tag{4.19}$$

After fixing the κ-symmetry as[5]

$$(1+\Gamma_{\hat{t}\hat{1}\hat{2}})\theta=0\,, \tag{4.20}$$

the Wess-Zumino term becomes

$$\int B=-\int d\xi^3\,\bar{\theta}\Gamma^{\hat{\mu}}(\partial_\mu+\Omega_\mu)\theta+\mathcal{O}(\epsilon^3)\,. \tag{4.21}$$

Writing the 11D gamma matrices in terms of the 16×16 euclidean 9D gamma matrices[6] and $\gamma^0=-1$, (A.1), the κ-symmetry fixing condition, (4.20), can be solved by a 16-component 9D Majorana spinor, ψ,

$$\theta=\frac{1}{2\sqrt{2}}\begin{pmatrix}\psi\\ -\gamma^{12}\psi\end{pmatrix}\,,\qquad \bar{\theta}=\frac{i}{2\sqrt{2}}(-\psi^\dagger\gamma^{12}\,,\psi^\dagger)\,. \tag{4.22}$$

To express the final form of the action in terms of the finite quantities, we replace the transverse coordinates,

$$(x^l,\,y,\,\psi)\longrightarrow 2\pi l_s^2(\phi^l,\,\phi^y,\,\psi)\,,\qquad l=3,\ldots,9\,. \tag{4.23}$$

Now, in the low energy limit, the supermembrane action reduces to, with $L=3,\ldots,9,y$,

$$S_{M2}=-T_{M2}\int d\xi^3\;1+ \tag{4.24}$$

$$+\frac{1}{(g_s/l_s)}\int d\xi^3\left[\begin{array}{l}-\frac{1}{2}\partial_\mu\phi^L\partial^\mu\phi^L-i\frac{1}{2}\psi^\dagger\gamma^\mu\partial_\mu\psi-\\ -\frac{\mu}{3\sqrt{2}}(\phi^4\partial_1\phi^y+\phi^3\partial_2\phi^y)+i\frac{\mu}{24\sqrt{2}}\psi^\dagger(\gamma^{24}-\gamma^{13}+3\gamma^{789})\psi-\\ -\frac{1}{2}(\frac{\mu}{6\sqrt{2}})^2(\phi_5^2+\phi_6^2)-\frac{1}{2}(\frac{\mu}{3\sqrt{2}})^2(\phi_3^2+\phi_4^2+\phi_7^2+\phi_8^2+\phi_9^2)\end{array}\right].$$

The dualization of this action to a U(1) gauge theory is performed in the subsection 4.4.

4.3 Derivation from the D2 Dirac-Born-Infeld action

In this subsection, we derive the massive gauge theory as a low energy limit of the Dirac-Born-Infeld action. As the explicit form of the supersymmetric DBI action in terms of the component fields is not known in the generic background or the pp-wave background,[7] we focus on the bosonic sector.

Writing the eleven dimensional pp-wave geometry, (2.28), as

$$ds_{11}^2=e^{-2\phi/3}ds_{IIA}^2+e^{4\phi/3}(dy+C_{(1)})^2\,, \tag{4.25}$$

[5]An identical gauge choice in the string case was considered in [42] and called "physical gauge".

[6]Here, for simplicity, we drop the hat symbol for the flat spacetime index in the 9D gamma matrix.

[7]For the superfield formalism, see [43, 44].

and compactifying the y direction, we obtain the ten dimensional type IIA supergravity background,

$$\begin{aligned} ds^2_{IIA} &= -\left(1-\frac{\tilde{H}}{2}\right)^{-1/2}\left(dt-\frac{\mu}{3\sqrt{2}}x^4dx^1-\frac{\mu}{3\sqrt{2}}x^3dx^2\right)^2+\left(1-\frac{\tilde{H}}{2}\right)^{1/2}\sum_{n=1}^{9}dx^ndx^n\,, \\ e^{\phi} &= \left(1-\frac{\tilde{H}}{2}\right)^{3/4}\,. \end{aligned} \tag{4.26}$$

In the limit, $x^M \sim \epsilon \to 0$, $M = 3, 4, \ldots$, we have been taking, the dilaton is real and small. The non-vanishing components of the RR one form, $C_{(1)} = C_M dx^M$, are

$$\begin{aligned} C_t &= -\frac{\tilde{H}}{2}\left(1-\frac{\tilde{H}}{2}\right)^{-1} = \mathcal{O}(\epsilon^2)\,, \\ C_1 &= \frac{\mu}{3\sqrt{2}}x^4\left(1-\frac{\tilde{H}}{2}\right)^{-1} = \frac{\mu}{3\sqrt{2}}x^4+\mathcal{O}(\epsilon^2)\,, \\ C_2 &= \frac{\mu}{3\sqrt{2}}x^3\left(1-\frac{\tilde{H}}{2}\right)^{-1} = \frac{\mu}{3\sqrt{2}}x^3+\mathcal{O}(\epsilon^2)\,, \end{aligned}$$

while the three form and four form fluxes are

$$H_{789} = (dB^{IIA})_{789} = \frac{\mu}{\sqrt{2}}\,, \qquad F_{t789} = (dC_{(3)})_{t789} = \frac{\mu}{\sqrt{2}}\,. \tag{4.27}$$

A few comments are in order. The resulting 10D background breaks all the supersymmetries in the type IIA supergravity, since no constraint on the constant spinor, η_0, removes the y dependence from the 11D Killing spinor expression, (3.17). From (3.18), the periodic identification over the y direction is compatible with the Killing spinors, (3.17), only for the special values of the compactification radii, e.g. zero [45].

The action describing the D2-brane consists of the Dirac-Born-Infeld and the Wess-Zumino terms [46],

$$\begin{aligned} S_{D2} &= S_{DBI}+S_{WZ}\,, \\ S_{DBI} &= -T_{D2}\int d\xi^3\, e^{-\phi}\sqrt{-\det(h_{\mu\nu}+\mathcal{F}_{\mu\nu})}\,, \\ S_{WZ} &= T_{D2}\int\left(C_{(1)}\wedge\mathcal{F}+C_{(3)}\right)\,, \end{aligned} \tag{4.28}$$

where $T_{D2} = 1/(4\pi^2 g_s l_s^3)$, and

$$h_{\mu\nu} = \partial_\mu x^M \partial_\nu x^N g^{IIA}_{MN}\,, \qquad \mathcal{F}_{\mu\nu} = 2\pi l_s^2 F_{\mu\nu} - \partial_\mu x^M \partial_\nu x^N B^{IIA}_{MN}\,. \tag{4.29}$$

Now adopting the static gauge, (3.21), replacing the transverse coordinates, x^l, by $2\pi l_s^2\phi^l$, $l = 3, \ldots, 9$, and taking the limit, $l_s^2 \sim \epsilon \to 0$ while keeping $g^2_{YM} = g_s/l_s$ finite, the terms involving B^{IIA} and $C_{(3)}$ vanish. In this low energy limit, the above D2-brane action becomes

$$S_{D2} = -T_{D2}\int d\xi^3\, 1 + \tag{4.30}$$

$$+\frac{1}{(g_s/l_s)}\int d\xi^3 \begin{bmatrix} -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi^l\partial^\mu\phi^l + \frac{\mu}{3\sqrt{2}}(\phi^4F_{02} - \phi^3F_{01}) - \\ -\frac{1}{2}(\frac{\mu}{6\sqrt{2}})^2(\phi_5^2+\phi_6^2) - \frac{1}{2}(\frac{\mu}{3\sqrt{2}})^2(\phi_7^2+\phi_8^2+\phi_9^2) \end{bmatrix}. \tag{4.31}$$

Although the precise form of the non-abelian Dirac-Born-Infeld action is not known (cf. [47]), the non-abelian generalization of the above bosonic quadratic action can be done following the Myers' prescription [48], which will result in the bosonic part of (1.2).[8] Contrary to the supermembrane case, all the terms linear in μ, including the Myers term, arise from the Wess-Zumino term.

4.4 Mutual agreement among the results through the dualization

In this subsection, we compare the resulting three actions, $\mathcal{S}$ from the matrix model (1.2), S_{M2} from the supermembrane (4.24), and S_{D2} from the D2-brane (4.30). By tuning the gauge choices in each setup to the consistent one, we show that all the actions agree with another.

Before starting, we justify the scaling, (4.13), we took in the last step of the derivation of the action, $\mathcal{S}$, in the matrix model setup. The scaling of the field variables is merely a field redefinition, while that of the worldvolume coordinates is taken to make the choice of the "time" coordinate in the matrix model consistent with the static gauge in the M2/D2 action,

$$\xi^0 = x^+ \quad\longrightarrow\quad \xi^0 = \sqrt{2}x^+ = x^0 + x^{10} \;\sim\; x^0\,, \tag{4.32}$$

since the compactification radius, R_y, is vanishingly small and $0 \le y = x^{10} < 2\pi R_y \to 0$.

It is worth to note that, although the periodic identification over the y direction is not compatible with the 11D Killing spinors for the generic values of the radii, in the small radius limit, the compactified pp-wave geometry may well recover the full supersymmetries. One way to understand this is going back to the light-cone coordinates, $x^\pm = (t \pm y)/\sqrt{2}$, which are periodic as $x^\pm \sim x^\pm \pm 2\pi(R_y/\sqrt{2})$. We take the infinite boost along the y direction such that the compactification over the y direction turns into that over the x^- direction of a finite radius. In the limit, the x^+ coordinate possesses no periodicity and serves the role of the "time" coordinate. Since the 11D Killing spinors, (3.17), are independent of x^-, no supersymmetry is broken under the compactification over the x^- direction. In fact, this was the basic setup the M-theory matrix model was originally obtained [15]–[18]. We also note that, in the same limit, the supersymmetric membrane configuration spanning the (x^0, x^1, x^2) directions in the probe analysis can be identified with the one spanning the (x^+, x^1, x^2) directions in the matrix model.

[8] However, in general, there is an ambiguity when one tries to do the non-abelian generalization. One can put an arbitrary numerical factor, say λ, in front of any commutator. The appropriate scaling of the fields like $A_\mu \to A_\mu/\lambda$, may absorb the numerical factor, but alters the string length in (4.29) as $l_s \to l_s/\sqrt{\lambda}$. Hence, different choices are physically distinct. Unfortunately, we are not able to fix the value in our framework, but set $\lambda = 1$ in (1.2) for simplicity.

Now we identify S_{M2} with S_{D2} through the dualization of the gauge fields to the compact scalar,[9] ϕ^y. We add a total derivative term to S_{M2},

$$S_{M2} \longrightarrow S_{M2} + \frac{1}{(g_s/l_s)}\int d\xi^3 \; \epsilon^{\mu\nu\rho}\partial_\mu\phi^y\partial_\nu A_\rho \,, \tag{4.33}$$

and integrate out the scalar. Effectively this replaces the derivatives of ϕ^y in the right hand side of (4.33) by

$$\partial_0\phi^y = -F_{12}\,, \qquad \partial_1\phi^y = F_{20} - \frac{\mu}{3\sqrt{2}}\phi^4\,, \qquad \partial_2\phi^y = F_{01} - \frac{\mu}{3\sqrt{2}}\phi^3\,, \tag{4.34}$$

which results in the supersymmetric completion of S_{D2},

$$\mathcal{S}' = \frac{1}{(g_s/l_s)}\int d\xi^3 \begin{bmatrix} -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi^l\partial^\mu\phi^l - i\frac{1}{2}\psi^\dagger\gamma^\mu\partial_\mu\psi + \\ +\frac{\mu}{3\sqrt{2}}(\phi^4F_{02} - \phi^3F_{01}) + i\frac{\mu}{24\sqrt{2}}\psi^\dagger(\gamma^{24} - \gamma^{13} + 3\gamma^{789})\psi - \\ -\frac{1}{4}\left(\frac{\mu}{6}\right)^2(\phi_5^2 + \phi_6^2) - \frac{1}{4}\left(\frac{\mu}{3}\right)^2(\phi_7^2 + \phi_8^2 + \phi_9^2) \end{bmatrix}, \tag{4.35}$$

where $l = 3, 4, \ldots, 9$.

Finally, we match $\mathcal{S}'$ with the abelian version of $\mathcal{S}$. As done in the matrix model setup, (2.2) and (2.5), we rotate the scalars and the fermion, ϕ^5, ϕ^6, ψ in $\mathcal{S}'$ such that the mass terms for the scalars disappear and that for the fermion gets modified. As stated earlier, this removes the explicit time dependency in the worldvolume supersymmetry transformations. The resulting action is of the same form as $\mathcal{S}$, except the $\pi/2$ rotation of the worldvolume coordinates,

$$(\xi^0,\,\xi^1,\,\xi^2) \longrightarrow (\xi^0,\,-\xi^2,\,\xi^1)\,, \tag{4.36}$$

which accompanies $(\gamma^1, \gamma^2) \to (-\gamma^2, \gamma^1)$. This $\pi/2$ rotation is an artifact of the two different gauge choices taken in the matrix model and in the supergravity analysis, since, in the matrix model setup, we choose the worldvolume coordinates as $X^1 = -\xi^2/\theta$, $X^2 = \xi^1/\theta$, (3.1) and (3.2). After the rotation, $\mathcal{S}'$, is exactly mapped to the abelian part of $\mathcal{S}$.

5. Worldvolume supersymmetry

In this section, we derive the worldvolume supersymmetries from the matrix model and from the supergravity, respectively.

5.1 Worldvolume SUSY from the matrix model

As seen in (4.11), the dynamical supersymmetry transformation of the fermions becomes singular when we take the commutative limit, $\theta \to 0$. To remedy the problem, one should first impose the following constraints on the Killing spinors,

$$\gamma^{14}\mathcal{E} = \gamma^{23}\mathcal{E} = \gamma^{789}\mathcal{E}\,, \tag{5.1}$$

which in turn implies $\gamma^{56}\mathcal{E} = \gamma^{789}\mathcal{E}$, so that the time dependency of $\mathcal{E}(t)$, (4.12), effectively

[9] Note that, from $2\pi l_s^2\phi^y = y$ and $R_y = g_s l_s$, in the low energy limit, $g_s \sim l_s \sim \epsilon^{1/2} \to 0$, the periodicity of ϕ^y is finite, g_s/l_s.

disappears. The only remaining singular part is now $(1/\theta)\gamma_{12}\mathcal{E}$, and this can be removed by adding the kinematical supersymmetry given by $\mathcal{E}' = -(1/\theta)\gamma_{12}\mathcal{E}$. Again the time dependency of the kinematical supersymmetry transformation drops out. Therefore, the unbroken supersymmetries of the membranes reappear precisely as the supersymmetry of the worldvolume theory. As stated before, the whole constraints on the Killing spinors, (5.1), can be rewritten in a concise manner, using the projection matrix (3.11),

$$\Omega\mathcal{E} = \mathcal{E}\,. \tag{5.2}$$

The worldvolume supersymmetry transformations of the action, $\mathcal{S}$, (1.2), are then

$$\begin{aligned} \delta A_\mu &= i\psi^\dagger\gamma_\mu\mathcal{E}\,, \qquad \delta\phi_a = i\psi^\dagger\gamma_a\mathcal{E}\,, \\ \delta\psi &= \Big[\frac{1}{2}F_{\mu\nu}\tilde{\gamma}^\mu\gamma^\nu + D_\mu\phi_a\tilde{\gamma}^\mu\gamma^a - i\frac{1}{2}[\phi_a,\phi_b]\gamma^{ab} + \frac{\mu}{3\sqrt{2}}(\phi_p\gamma^p - \phi_r\gamma^r)\gamma^{789}\Big]\mathcal{E}\,, \end{aligned} \tag{5.3}$$

where $p = 5,6$, $r = 7,8,9$ and $\mathcal{E}$ is a time independent constant Majorana spinor subject to (5.2).

It is interesting to compare with the ordinary BFSS matrix model or the $\mu = 0$ case. In that case, the only singular piece in the $\theta \to 0$ limit of the dynamical supersymmetry transformation is $(1/\theta)\gamma_{12}\mathcal{E}$, and this can be completely removed by the kinematical supersymmetry transformation. Thus, both in the $\mu = 0$ and $\mu \neq 0$ cases, the commutative worldvolume actions possess the same numbers of supersymmetries the background membranes preserve, i.e. 16 for $\mu = 0$, and 4 for $\mu \neq 0$.

5.2 Worldvolume SUSY from the supermembrane action

In this subsection, we derive the worldvolume supersymmetry transformations of the quadratic actions, S_{M2} (4.24) and $\mathcal{S}'$ (4.35). The worldvolume supersymmetry is identified as a specific combination of the spacetime supersymmetry and the κ-symmetry,

$$\delta\theta = \eta + (1+\Gamma)\kappa\,, \qquad \delta x^M = \bar{\theta}\Gamma^M\eta - \bar{\theta}\Gamma^M(1+\Gamma)\kappa\,, \qquad M = 0,1,\dots,9,y\,, \tag{5.4}$$

which must preserve the κ-symmetry fixing, (4.20), as well as the static gauge, (3.21) [35, 39, 42],

$$(1+\Gamma_{\hat{t}\hat{1}\hat{2}})\delta\theta = 0\,, \tag{5.5}$$

$$\delta x^0 = \delta x^1 = \delta x^2 = 0\,. \tag{5.6}$$

In the case of the pp-wave background geometry we consider, the κ-symmetry parameter, $\kappa(\xi)$, is an arbitrary fermionic 'local' variable, while the Killing spinor, η, is of the fixed form, (3.17), with an arbitrary 'constant' spinor, η_0. From (2.28), there exist translational isometries in the (x^0, x^1, x^2) directions. However, these rigid isometries serve no role to ensure the vanishing of the local transformations, (5.6).

In the limit, $(x^L, \theta) \sim \epsilon \to 0$, $L = 3,\dots,9,y$, the lagrangian terminates at the quadratic order in ϵ, and the worldvolume supersymmetry transformations are to be kept up to the linear order. From (2.23), (3.17), (4.20), $\bar{\theta}(1+\Gamma_{\hat{t}\hat{1}\hat{2}})=0$ and

$$\begin{aligned}
\delta\theta &= (1+\Gamma_{\hat{t}\hat{1}\hat{2}})\kappa_0(\xi) + (1-\xi^1\Omega_1-\xi^2\Omega_2)e^{-t\Omega_t}\eta_0 + \\
&\quad +(1+\Gamma_{\hat{t}\hat{1}\hat{2}})\kappa_1(\xi) + \partial_\mu x^L \Gamma_{\hat{L}}\Gamma^{\hat{\mu}}\Gamma_{\hat{t}\hat{1}\hat{2}}\kappa_0(\xi) - \left(\sum_{n=3}^{9} x^n\Omega_n + y\Omega_t\right) e^{-t\Omega_t}\eta_0 + \mathcal{O}(\epsilon^2)\,, \\
\delta x^\mu &= \bar{\theta}\Gamma^\mu(1-\xi^1\Omega_1-\xi^2\Omega_2)e^{-t\Omega_t}\eta_0 + \mathcal{O}(\epsilon^2)\,,
\end{aligned} \tag{5.7}$$

where $L=3,\ldots,9,y$ and $\kappa_0(\xi),\kappa_1(\xi)$ denote the zeroth, first order of $\kappa(\xi)$ in ϵ.

Imposing the constraint, (5.5), one can solve for $(1+\Gamma_{\hat{t}\hat{1}\hat{2}})\kappa_0$, $(1+\Gamma_{\hat{t}\hat{1}\hat{2}})\kappa_1$, and the vanishing of δx^μ is equivalent to

$$(1-\Gamma_{\hat{t}\hat{1}\hat{2}})(1-\xi^1\Omega_1-\xi^2\Omega_2)e^{-t\Omega_t}\eta_0 = 0\,. \tag{5.8}$$

This relation must hold for arbitrary ξ^1,ξ^2,t so that, from (3.19), we get the same constraints on η_0 as in the probe analysis, (3.24),

$$\Gamma_{\hat{t}\hat{1}\hat{2}}\eta_0 = \Gamma^{\hat{7}\hat{8}\hat{9}\hat{4}\hat{1}}\eta_0 = \Gamma^{\hat{7}\hat{8}\hat{9}\hat{3}\hat{2}}\eta_0 = \eta_0\,. \tag{5.9}$$

After all, from (4.23), in terms of the 16-component spinors, ψ and $\mathcal{E}$, of which the latter gives the solution of $(1-\Gamma_{\hat{t}\hat{1}\hat{2}})\eta_0=0$,

$$\eta_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}\mathcal{E}\\ \gamma^{12}\mathcal{E}\end{pmatrix}, \tag{5.10}$$

the worldvolume supersymmetry transformations read

$$\begin{aligned}
\delta\phi^l &= i\psi^\dagger\gamma^l\mathcal{E}(t)\,, \qquad \delta\phi^y = -i\psi^\dagger\gamma^{12}\mathcal{E}(t)\,, \qquad l=3,4,\ldots 9\,, \\
\delta\psi &= \left[-\partial_\mu\phi^l\gamma_l\gamma^\mu - \partial_\mu\phi^y\tilde{\gamma}^\mu\gamma^{12} + \frac{\mu}{3\sqrt{2}}(x^3\gamma^1 - x^4\gamma^2) - \frac{\mu}{12\sqrt{2}}\phi^p(\gamma_p\gamma^{789} + 3\gamma^{789}\gamma_p)\right]\mathcal{E}(t)\,, \\
\mathcal{E}(t) &\equiv e^{-\frac{\mu}{12\sqrt{2}}\gamma^{789}t}\mathcal{E}\,, \quad \gamma^{24}\mathcal{E} = \gamma^{31}\mathcal{E} = \gamma^{789}\mathcal{E}\,, \quad \mathcal{E} = C\mathcal{E}^*\,, \qquad p=5,6,\ldots,9\,.
\end{aligned} \tag{5.11}$$

From the dual relation, (4.34), one can also obtain the supersymmetry transformations of the quadratic D2 action, $\mathcal{S}'$, (4.35),

$$\begin{aligned}
\delta\phi^l &= i\psi^\dagger\gamma^l\mathcal{E}(t)\,, \\
\delta F_{\mu\nu} &= i\partial_\mu[\psi^\dagger\gamma_\nu\mathcal{E}(t)] - i\partial_\nu[\psi^\dagger\gamma_\mu\mathcal{E}(t)] + \mathcal{O}(\psi)\,, \\
\delta\psi &= \left[\frac{1}{2}F_{\mu\nu}\tilde{\gamma}^\mu\gamma^\nu + \partial_\mu\phi^l\tilde{\gamma}^\mu\gamma_l - \frac{\mu}{12\sqrt{2}}\phi^p(\gamma_p\gamma^{789} + 3\gamma^{789}\gamma_p)\right]\mathcal{E}(t)\,,
\end{aligned} \tag{5.12}$$

where $l=3,\ldots,9$, $p=5,\ldots,9$, and $\mathcal{O}(\psi)$ denotes the terms which vanish when we impose the equation of motion for ψ. Such terms are there since we integrated out ϕ^y using its equation of motion. Nevertheless, the off-shell supersymmetry transformations of the action, $\mathcal{S}'$, are given by the above formulae without $\mathcal{O}(\psi)$. Finally, as done in the subsection 4.4, tuning the gauge choices, one can show that the above worldvolume supersymmetry is consistent with the one derived in the matrix model, (5.1), (5.3).

5.3 Supersymmetry algebra and the supermultiplets

The supersymmetry algebra of the action, $\mathcal{S}$, (1.2), can be read off easily from our previous work on the five dimensional theory [11], through the dimensional reduction. The supersymmetry algebra of the 3D $\mathcal{N}=2$ worldvolume theory reads, with the hamiltonian, H, so(2), so(3) R-symmetry generators, M_{56}, M_{rs}, and real central charges, $\mathcal{R}$, $\mathcal{R}_r$, $\mathcal{A}_r$, $\mathcal{B}_r$, $r=7,8,9$,

$$[H,Q] = 0\,, \tag{5.13}$$

$$[M_{56},Q] = i\frac{1}{2}\gamma_{56}Q\,, \qquad [M_{rs},Q] = i\frac{1}{2}\gamma_{rs}Q\,,$$

$$[M_r,M_s] = i\epsilon_{rst}M_t\,, \qquad M_r = \frac{1}{2}\epsilon_{rst}M_{st}\,, \tag{5.14}$$

$$\{Q,\,Q^\dagger\} = 2\Omega\Big[H-\mathcal{R}-\frac{\mu}{3\sqrt{2}}M_{56}+\gamma^r\left(\mathcal{R}_r+\frac{\mu}{3\sqrt{2}}M_r\right)+\gamma^{125r}\mathcal{A}_r+\gamma^{436r}\mathcal{B}_r\Big]\Omega\,. \tag{5.15}$$

The supercharge is subject to

$$Q = C(Q^\dagger)^T\,, \qquad Q = \Omega Q\,, \tag{5.16}$$

resulting in the four independent real components. The explicit forms of H, M_{56}, M_{rs}, $\mathcal{R}$, $\mathcal{R}_r$, $\mathcal{A}_r$, $\mathcal{B}_r$ are given in the appendix A.2. The numbers of degrees in the left and right hand sides of (5.15) match as

$$10 = 1+3+3+3\,. \tag{5.17}$$

Note that Ω, $\Omega\gamma^r\Omega$, $\Omega\gamma^{125r}\Omega$, $\Omega\gamma^{436r}\Omega$ are the only allowed independent gamma matrix products to appear on the right. From the positive definity, we have the following BPS energy bound,

$$H \geq \mathcal{R}+\frac{\mu}{3\sqrt{2}}M_{56}+\left|(\hat{e}_1)_r\left(\mathcal{R}_r+\frac{\mu}{3\sqrt{2}}M_r\right)\right|+|(\hat{e}_2)_r\mathcal{A}_r|+|(\hat{e}_3)_r\mathcal{B}_r|\,, \tag{5.18}$$

where $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ form an arbitrary orthonormal real basis for the "$7,8,9$" space so that $(\hat{e}_1)_r\gamma^r$, $(\hat{e}_2)_r\gamma^{125r}$, $(\hat{e}_3)_r\gamma^{436r}$ can be simultaneously diagonalized with the eigenvalues, ± 1.

The energy spectra and the numbers of the corresponding bosons and fermions can be obtained by solving the abelian sector of the equations of motion, (A.11). They are summarized in table 1. Each row forms an independent supermultiplet, and there exit three multiplets. Note that in three dimensions the gauge fields have only one on-shell degree. In the present massive gauge theory, the nontrivial linear combinations of the gauge fields and the two Higgs, ϕ_3, ϕ_4, form three independent degrees, one for each multiplet.

5.4 BPS equations for the fully supersymmetric configurations and vacua

In this subsection, we consider the BPS equations which describe the configurations preserving all the four supersymmetries. In the conventional supersymmetric models, such fully supersymmetric configurations would be vacua, but in the present case, the novel structure of the supersymmetry algebra allows nontrivial fully supersymmetric BPS configurations. They have the energy saturation,

$$H = \mathcal{R}+\frac{\mu}{3\sqrt{2}}M_{56}\,, \tag{5.19}$$

while other central and R-symmetry charges vanish, $\mathcal{R}_r=\mathcal{A}_r=\mathcal{B}_r=M_r=0$.

energy spectra	ψ	A_μ, ϕ_3, ϕ_4	ϕ_5, ϕ_6	ϕ_7, ϕ_8, ϕ_9
$E_{\mathbf{k}} = \sqrt{(\mu/3)^2 + \mathbf{k}^2}$	4	1	0	3
$E^+_{\mathbf{k}} = \sqrt{(\mu/6)^2 + \mathbf{k}^2} + \lvert\mu\rvert/6$	2	1	1	0
$E^-_{\mathbf{k}} = \sqrt{(\mu/6)^2 + \mathbf{k}^2} - \lvert\mu\rvert/6$	2	1	1	0

Table 1: Energy spectra and the numbers of bosons and fermions.

The corresponding BPS equations can be obtained either by writing $H - \mathcal{R} - \frac{\mu}{3\sqrt{2}} M_{56}$ as a sum of squares or from the supersymmetry transformation of the fermions [21, 38]. The BPS equations are

$$\begin{aligned}
&F_{0\mu} = D_0\phi_l = D_0\phi_r = 0\,, && D_0\phi_p - \frac{\mu}{3\sqrt{2}}\epsilon_{pq}\phi_q = 0\,, \\
&[\phi_r, \phi_s] - i\frac{\mu}{3\sqrt{2}}\epsilon_{rst}\phi_t = 0\,, && D_j\phi_r = [\phi_l, \phi_r] = [\phi_p, \phi_r] = 0\,, \\
&F_{12} - i[\phi_3, \phi_4] = 0\,, && D_1\phi_3 - D_2\phi_4 = 0\,, \\
&D_1\phi_4 + D_2\phi_3 - i[\phi_5, \phi_6] = 0\,, && D_j\phi_p + iJ_{jl}\epsilon_{pq}[\phi_l, \phi_q] = 0\,,
\end{aligned} \tag{5.20}$$

where $j = 1, 2$, $l = 3, 4$, $p = 5, 6$, $r = 7, 8, 9$, $J_{14} = J_{23} = \epsilon_{56} = 1$. The BPS equations themselves satisfy the Gauss constraint so that any BPS solution satisfies the full equations of motion, (A.11). The last four BPS equations are essentially the dimensional reduction of the BPS equations in the 6D euclidean pure super Yang-Mills [38].

The classical supersymmetric vacua are given by the constant fuzzy spheres and arbitrary vevs for ϕ_3, ϕ_4,

$$[\phi_r, \phi_s] = i\frac{\mu}{3\sqrt{2}}\epsilon_{rst}\phi_t\,, \qquad \phi_3 = c_3\,, \qquad \phi_4 = c_4\,, \qquad \phi_5 = \phi_6 = F_{\mu\nu} = 0\,. \tag{5.21}$$

From (4.34), after tuning of the gauge choices, the dual relation between the field strength and the compact scalar becomes

$$F_{12} = -\partial_0\phi^y\,, \qquad F_{20} = \partial_1\phi^y + \frac{\mu}{3\sqrt{2}}\phi^3\,, \qquad F_{01} = \partial_2\phi^y - \frac{\mu}{3\sqrt{2}}\phi^4\,. \tag{5.22}$$

Therefore, geometrically viewed from the eleven dimensions, the vacua correspond to the giant graviton plus the membranes tilted to the eleventh direction,

$$\phi^y = -\frac{\mu}{3\sqrt{2}}c_3x^1 + \frac{\mu}{3\sqrt{2}}c_4x^2\,. \tag{5.23}$$

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A. Conventions and useful formulae

A.1 In the supergravity setup

To make a connection to the matrix model, we choose the following representation of the flat eleven dimensional spacetime gamma matrices,

$$\Gamma^{\hat{S}} = \begin{pmatrix} 0 & \tilde{\gamma}^{\hat{S}} \\ \gamma^{\hat{S}} & 0 \end{pmatrix}, \qquad \Gamma^{\hat{y}} = \Gamma^{\hat{0}\hat{1}\cdots\hat{9}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.1}$$

where $\hat{S} = 0, 1, \ldots, 9$ and, in terms of the euclidean nine dimensional gamma matrices, γ^A, $A = 1, 2, \ldots, 9$,

$$\tilde{\gamma}^{\hat{R}} = (1, \gamma^A), \qquad \gamma^{\hat{R}} = (-1, \gamma^A). \tag{A.2}$$

Thus, in terms of the 9D euclidean charge conjugate matrix, C, (2.10), the eleven dimensional complex conjugate matrix, $\mathcal{B}$, is written as

$$\mathcal{B} = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix}, \qquad \left(\Gamma^{\hat{R}}\right)^* = \mathcal{B}^{-1}\Gamma^{\hat{R}}\mathcal{B}, \qquad \hat{R} = 0, 1, \ldots, 10. \tag{A.3}$$

The 32-component 11D Majorana spinor, θ, satisfies

$$\theta = \mathcal{B}\theta^*. \tag{A.4}$$

We take the vielbein of the pp-wave metric, (2.28), as follows,

$$\begin{aligned}
e^{\hat{t}} &= \left(1 + \frac{H}{2}\right)^{1/2}\left(dt + \frac{H/2}{1 + H/2}dy\right), \qquad e^{\hat{y}} = \left(1 + \frac{H}{2}\right)^{-1/2} dy, \\
e^{\hat{1}} &= dx^1 + \frac{\mu}{3\sqrt{2}}x^4(dt + dy), \qquad e^{\hat{2}} = dx^2 + \frac{\mu}{3\sqrt{2}}x^3(dt + dy), \\
e^{\hat{l}} &= dx^l, \qquad l = 3, \ldots, 9,
\end{aligned} \tag{A.5}$$

from which the non-vanishing spin connections can be determined,

$$\begin{aligned}
\omega_{\hat{t}\hat{y}\,l} &= -\frac{1}{4}\left(1 + \frac{H}{2}\right)^{-1}\partial_l H, \qquad l = 3, \ldots, 9, \\
\omega_{\hat{t}\hat{l}\,t} &= \omega_{\hat{t}\hat{l}\,y} = \omega_{\hat{y}\hat{l}\,y} = \omega_{\hat{y}\hat{l}\,t} = \frac{1}{4}\left(1 + \frac{H}{2}\right)^{-1/2}\partial_l H - \left(\frac{\mu}{6}\right)^2\left(1 + \frac{H}{2}\right)^{-1/2}(x^3\delta_{\hat{l}\hat{3}} + x^4\delta_{\hat{l}\hat{4}}), \\
\omega_{\hat{t}\hat{1}\,4} &= \omega_{\hat{t}\hat{4}\,1} = \omega_{\hat{t}\hat{2}\,3} = \omega_{\hat{t}\hat{3}\,2} = \omega_{\hat{y}\hat{1}\,4} = \omega_{\hat{y}\hat{4}\,1} = \omega_{\hat{y}\hat{2}\,3} = \omega_{\hat{y}\hat{3}\,2} = -\frac{\mu}{6\sqrt{2}}\left(1 + \frac{H}{2}\right)^{-1/2}, \\
\omega_{\hat{1}\hat{4}\,t} &= \omega_{\hat{1}\hat{4}\,y} = \omega_{\hat{2}\hat{3}\,t} = \omega_{\hat{2}\hat{3}\,y} = -\frac{\mu}{6\sqrt{2}}.
\end{aligned} \tag{A.6}$$

The explicit forms of the curved spacetime gamma matrices are

$$\begin{aligned}
\Gamma_t &= \left(1 + \frac{H}{2}\right)^{1/2}\Gamma_{\hat{t}} + \frac{\mu}{3\sqrt{2}}x^4\Gamma_{\hat{1}} + \frac{\mu}{3\sqrt{2}}x^3\Gamma_{\hat{2}}, \\
\Gamma_y &= \left(1 + \frac{H}{2}\right)^{-1/2}\Gamma_{\hat{y}} + \frac{H}{2}\left(1 + \frac{H}{2}\right)^{-1/2}\Gamma_{\hat{t}} + \frac{\mu}{3\sqrt{2}}x^4\Gamma_{\hat{1}} + \frac{\mu}{3\sqrt{2}}x^3\Gamma_{\hat{2}}, \\
\Gamma_1 &= \Gamma_{\hat{1}}, \qquad \Gamma_2 = \Gamma_{\hat{2}}, \qquad \Gamma_l = \Gamma_{\hat{l}}, \quad l = 3, \ldots, 9.
\end{aligned} \tag{A.7}$$

In the given pp-wave background geometry, (2.28), the supersymmetry variations of the gravitino reduce to, with $\partial_\pm \equiv \frac{1}{\sqrt{2}}(\partial_t \pm \partial_y)$, $n = 1, \ldots, 9$,

$$
\begin{aligned}
\frac{1}{\sqrt{2}}(\delta\psi_t + \delta\psi_y) &= \Bigg[\partial_+ + \Omega_+ - \frac{1}{4}\partial_n H \left(1+\frac{H}{2}\right)^{-1/2}\Gamma^{\hat{+}\hat{n}} + \\
&\qquad + \frac{\mu}{3}\left(1+\frac{H}{2}\right)^{-1/2}(x^3\Omega_2 + x^4\Omega_1)\Bigg]\eta\,, \\
\frac{1}{\sqrt{2}}(\delta\psi_t - \delta\psi_y) &= \partial_-\eta\,, \qquad \delta\psi_n = \left[\partial_n + \frac{1}{8}\partial_n H\left(1+\frac{H}{2}\right)^{-1}\Gamma^{\hat{t}\hat{y}} + \left(1+\frac{H}{2}\right)^{-1/2}\Omega_n\right]\eta\,.
\end{aligned}
$$

Some useful relations to derive the Killing spinors, (3.17), are

$$
[\Omega_1, \Omega_+] = [\Omega_2, \Omega_+] = 0\,, \tag{A.8}
$$

$$
[\Omega_l, \Omega_+] = \begin{cases} \dfrac{\mu}{3}\Omega_l \Gamma^{\hat{7}\hat{8}\hat{9}}\,, & l = 3, 4\,, \\ \dfrac{\mu^2}{72}\Gamma^{\hat{+}\hat{l}}\,, & l = 5, 6\,, \\ \dfrac{\mu^2}{18}\Gamma^{\hat{+}\hat{l}}\,, & l = 7, 8, 9\,. \end{cases} \tag{A.9}
$$

A.2 In the 3D $\mathcal{N} = 2$ massive super Yang-Mills action

The explicit forms of the supercharge, Q, hamiltonian, H, so(2), so(3) R-symmetry generators, M_{56}, M_{rs}, and real central charges, $\mathcal{R}$, $\mathcal{R}_r$, $\mathcal{A}_r$, $\mathcal{B}_r$ are

$$
\begin{aligned}
Q &= \Omega \int dx^2\, \mathrm{tr}_N \left[-\frac{1}{2}F_{\mu\nu}\gamma^\mu\tilde{\gamma}^\nu + D_\mu\phi_a\gamma^a\tilde{\gamma}^\mu + i\frac{1}{2}[\phi_a, \phi_b]\gamma^{ab} + \frac{\mu}{3\sqrt{2}}\phi_a\gamma^a\gamma^{789}\right]\psi\,, \\
H &= \int dx^2\ \mathrm{tr}_N \Bigg[\frac{1}{2}F_{0i}^2 + \frac{1}{2}F_{12}^2 + \frac{1}{2}D_0\phi_a^2 + \frac{1}{2}D_i\phi_a^2 - \frac{1}{4}[\phi_a, \phi_b]^2 + i\frac{\mu}{3\sqrt{2}}\epsilon^{rst}\phi_r\phi_s\phi_t + \\
&\qquad + \frac{1}{2}\left(\frac{\mu}{3\sqrt{2}}\right)^2\phi_r^2\Bigg]\,, \\
M_{56} &= \int dx^2\ \mathrm{tr}_N \left[\epsilon^{pq}D_0\phi_p\phi_q - \frac{\mu}{6\sqrt{2}}(\phi_5^2 + \phi_6^2) - i\frac{1}{4}\psi^\dagger\gamma_{56}\psi\right]\,, \\
M_{rs} &= \int dx^2\ \mathrm{tr}_N \left[D_0\phi_r\phi_s - D_0\phi_s\phi_r - i\frac{1}{4}\psi^\dagger\gamma_{rs}\psi\right]\,, \\
\mathcal{R} &= \int dx^2\, \partial_i\ \mathrm{tr}_N \left(\frac{1}{2}\epsilon^{ij}(\phi_3 D_j\phi_4 - \phi_4 D_j\phi_3) - iJ^{il}\phi_5[\phi_l, \phi_6]\right)\,, \\
\mathcal{R}_r &= -i\frac{1}{2}\epsilon_{rst}\int dx^2\, \partial_i\ \mathrm{tr}_N \left(J^{il}[\phi_l, \phi_s]\phi_t\right)\,, \\
\mathcal{A}_r &= -\int dx^2\, \partial_i\, \epsilon^{ij}\, \mathrm{tr}_N \left(\phi_r D_j\phi_5 + iJ_{jl}[\phi^l, \phi_6]\right)\,, \\
\mathcal{B}_r &= \int dx^2\, \partial_i\, \epsilon^{ij}\, \mathrm{tr}_N \left(\phi_r D_j\phi_6 - iJ_{jl}[\phi^l, \phi_5]\right)\,,
\end{aligned} \tag{A.10}
$$

where $a = 3, \ldots, 9$, $i = 1, 2$, $l = 3, 4$, $r = 7, 8, 9$ and $\epsilon^{12} = J^{14} = J^{23} = J_{14} = J_{23} = 1$.

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The equations of motion are

$$
\begin{aligned}
D_\nu F^\nu{}_0 + i[\phi_a, D_0\phi_a] + \frac{1}{2}\{\psi^{\dagger\alpha}, \psi_\alpha\} - \frac{\mu}{3\sqrt{2}}(D_1\phi_4 + D_2\phi_3 + i[\phi_5,\phi_6]) &= 0\,, \\
D_\nu F^\nu{}_i + i[\phi_a, D_i\phi_a] + \frac{1}{2}\{\psi^{\dagger\alpha}, (\gamma_i\psi)_\alpha\} - \frac{\mu}{3\sqrt{2}}J_{il}D_0\phi_l &= 0\,, \\
D_\mu D^\mu\phi_l - [\phi_a,[\phi_a,\phi_l]] + \frac{1}{2}\{\psi^{\dagger\alpha}, (\gamma_l\psi)_\alpha\} - \frac{\mu}{3\sqrt{2}}J_{li}F_{0i} &= 0\,, \\
D_\mu D^\mu\phi_p - [\phi_a,[\phi_a,\phi_p]] + \frac{1}{2}\{\psi^{\dagger\alpha}, (\gamma_p\psi)_\alpha\} + \frac{\mu}{3\sqrt{2}}\epsilon_{pq}D_0\phi_q &= 0\,, \\
D_\mu D^\mu\phi_r - [\phi_a,[\phi_a,\phi_r]] + \frac{1}{2}\{\psi^{\dagger\alpha}, (\gamma_r\psi)_\alpha\} - i\frac{\mu}{\sqrt{2}}\epsilon_{rst}\phi_s\phi_t - (\frac{\mu}{3\sqrt{2}})^2\phi_r &= 0\,, \\
\gamma^\mu D_\mu\psi - i\gamma^a[\phi_a,\psi] - \frac{\mu}{12\sqrt{2}}(\gamma^{14} + \gamma^{23} - \gamma^{56} + 3\gamma^{789})\psi &= 0\,, \quad \text{(A.11)}
\end{aligned}
$$

where $i = 1,2$, $l = 3,4$, $p = 5,6$, $r = 7,8,9$ and $J_{14} = J_{23} = 1$.

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